THE ANALYSIS OF SMALL WATER DISTRIBUTION SYSTEMS

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Water Resources Research Center

UNIVERSITY OF GUAM

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by

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Project Completion Report
for
A MODEL OF A PORTION OF THE
PUBLIC UTILITY AGENCY OF GUAM (PUAG) WATER DISTRIBUTION SYSTEM

OWRT Project No. A-006-Guam, Grant Agreement No. 14-34-001-7024
Principal Investigators: Stephen J. Winter and Robert D. Setter
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ABSTRACT

A method is derived for determining the flow lost per foot of pipe in situations involving pipes having leaks or illegal connections. The flow into the pipe and the pressure drop over the length of the pipe must be known. An example problem is solved.

A modification of the Hardy-Cross technique of pipe network analysis is presented which accounts for flow out of the network per foot of pipe. This modification can be used to account for unintentional loss as well as outflow to consumers in situations where an accurate description of the flow is required. This situation is most likely to arise in small distribution systems where the outflow to each consumer may be a significant part of the total flow in a pipe. Two example problems are solved, the second involving an expansion of the network considered in the first.
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INTRODUCTION

For all practical purposes, present day water distribution systems serving the civilian community of Guam were constructed subsequent to WWII. Prior to 1950, the beginning of civilian government on Guam, water distribution, like other utilities, was handled by the Navy government. Subsequent to that date, water distribution has been under the jurisdiction of the Public Utility Agency of Guam (PUAG).

During the latter phases of WWII, after American forces reoccupied Guam, a period of reconstruction ensued along with the development of water distribution systems in some of the larger cities. This development was hasty and rule of thumb design practices were often used. Records do not exist concerning the location of all pipes presently comprising these systems. Following this period, development was more orderly and records are available. However, in general, system development (and the search for new sources of potable water) has been the result of response to demand rather than the result of a master plan. This type of development has led to poor system performance in many areas, characterized by low pressures and, at worst, periods of no flow.

The problem becomes compounded because large quantities of water are lost in the distribution system (i.e., exist at the source but do not reach the consumer). Estimates of up to 30%
loss in the system have been made. There are at least two explanations for this. Firstly, leakage occurs, probably in older pipes. It is even possible that open pipes may exist, discharging their contents into the ground. Secondly, it is known that numerous illegal unmetered connections exist. Most have probably been made with equipment acquired illegally from PUAG.

OBJECTIVES

As originally planned, this project involved the construction of a physical model of a portion of the PUAG water distribution system. Serious consideration was later given to the possibility of constructing a digital computer model. However, difficulties in securing the metering devices required for obtaining field data and in locating a suitable portion of the system for analysis prevented the accomplishment of the original objectives.

As an alternative, the necessary theory has been developed and examples provided which illustrate how these objectives can be accomplished. Specifically, analytical procedures are presented whereby it is possible:

1. to locate areas of leakage and/or illegal connections, and
2. to determine the flow and pressure distribution within a portion of the system.
SCOPE AND METHOD

In case 1, pressure readings must be taken at fire hydrants or other convenient locations. Flow measurements must also be taken at one or more locations. By using this data as input to equations that will be derived in the Theory section of the report, it is possible to compute the flow out of the system per unit length of pipe. This flow can be compared with consumer meter readings in an effort to locate areas of loss. A numerical example of this type is worked in the Applications section of the report.

In case 2, the flows into and out of the system must be known. These can be obtained from flow measurements taken in the field and consumer meter readings. By using this data as input to an iterative technique that will be derived in the Theory section of the report, it is possible to determine the flow and pressure distribution in the portion of the system being analyzed. An example of this type is worked in the Applications section of the report. Furthermore, this technique makes it possible to predict the effects of alterations (expansions) of the system on its performance (i.e., the given input and output flows are modified in accordance with the expected increase in demand and changes in system performance can be observed, particularly areas of low pressure). An example of this type is also worked in the Applications section of the report.
The procedures which are presented are unique in that they account for all flow leaving pipes in a distribution system; outflows are not lumped together as is usually the case. This approach is especially valuable in the case of small distribution systems such as exist on Guam.

The procedures which are presented do not include provisions for pumping and/or storage. However, it will be seen that these items are simple to isolate from the portion of the system being considered.

THEORY

General Equations

The mathematical analysis of fluid flow in a network of pipes requires that three conditions be satisfied:

1. The algebraic sum of the flows, $Q$, at a junction must be zero.
2. The algebraic sum of the head losses, $H$, around a loop must be zero.
3. The proper relationship between head loss and flow must be satisfied for each pipe.

The first and second conditions are the well-known circuit laws and may be expressed as:

$$\sum Q = 0 \quad \ldots \quad 1$$
$$\sum H = 0 \quad \ldots \quad 2$$
Numerous relationships expressing the third condition exist. Because of simplicity, an exponential relationship is frequently used:

\[ H = kQ^n \] ... 3

where \( k \) is a constant that depends on pipe length, diameter, and condition; \( n \) is also a constant, approximately equal to 1.85, that depends on pipe condition. An alternate form of equation 3 is:

\[ H = k' Q^n \ell \] ... 4

where \( \ell \) is the length of the pipe being considered and \( k' \) is a constant that is independent of pipe length. (i.e., \( k' = k/\ell \)).

In case of small distribution systems, the flow to each consumer may be a significant part of the total flow in a given pipe of the network. Consequently, a method of accounting for all flow from each pipe is desirable. One way of achieving this objective is by means of a "distributed take-off" (similar to a distributed load on a beam), \( q(x) \), having units of flow/length. This concept is illustrated in Figure 1.
FIGURE 1. PARAMETERS FOR DISTRIBUTED TAKE-OFF.
The flow, \( Q_x \), at a point \( x \) in the pipe is

\[
q_x = Q_{in} - \int_0^x q(x) \, dx
\]

or, assuming that \( q(x) \) is a constant (i.e., \( q(x) = q \)),

\[
Q_x = Q_{in} - qx
\]

\( Q_{in} \) and \( Q_{out} \) are the flows into and out of the length of pipe, respectively. If the entire length of pipe is considered, then

\[
Q_{out} = Q_{in} - q\ell
\]

Using equation 4, the head loss occurring in a length of pipe \( dx \) at position \( x \) is

\[
dH' = k' Q_x^n \, dx
\]

where the prime following \( H \) denotes head loss in a pipe having distributed take-off. If \( Q \) varies as indicated by equation 5, then

\[
dH' = k' (Q_{in} - qx)^n \, dx
\]

Integrating over the length of the pipe,

\[
H' = k' \int_0^\ell (Q_{in} - qx)^n \, dx
\]

\[
= -\frac{k' (Q_{in} - qx)^{n+1}}{(n+1) q} \bigg|_0^\ell
\]

\[
= k' \frac{(Q_{in}^{n+1} - (Q_{in} - q\ell)^{n+1})}{(n+1) q}
\]

... 7
or, making use of equation 6,

\[ H' = k' \left( \frac{Q_{in}^{n+1} - Q_{out}^{n+1}}{(n+1)q} \right) \] ... 8

Equation 7 or 8 gives the head loss in a length of pipe having distributed take-off.

As \( q \) approaches zero, it is observed that equation 8 becomes indeterminate (i.e., approaches \( 0/0 \)). Using l'Hôpital's rule,

\[ \lim_{q \to 0} H' = \frac{k'}{n+1} \left[ \frac{d}{dq} \left( Q_{in}^{n+1} - \left( Q_{in} - q\ell \right)^{n+1} \right) \right]_{q \to 0} \]

\[ = \frac{k'}{n+1} \left[ - (n+1)(Q_{in} - q\ell)^n(-\ell) \right]_{q \to 0} \]

\[ = k' Q_{in} \ell \]

which is equivalent to equation 4. Thus, as expected, the head loss in a pipe having distributed take-off reduces to the case of a conventional pipe when \( q \) approaches zero. This is the maximum head loss that can occur in a pipe having distributed take-off since all the water must travel the full length of the pipe. Making note of this, the previous equation can be rewritten as

\[ H'_{\text{max}} = k' Q_{in} \ell \] ... 9
The minimum head loss occurs when the outflow, $Q_{out}$, approaches zero (i.e., none of the water travels the full length of the pipe) and $q$ is, therefore, maximum. Using equation 6,

$$Q_{out} = Q_{in} - q_{max} \ell = 0$$

or,

$$q_{max} = \frac{Q_{in}}{\ell}$$

Substituting this result into equation 8,

$$H'_{min} = k' \left( \frac{Q_{in}^{n+1} - (Q_{in} - (Q_{in}/\ell) \ell)^{n+1}}{(n+1)(Q_{in}/\ell)} \right)$$

$$= \frac{k' Q_{in}^{n} \ell}{n+1}$$

In summary, the lower limit on $q$, $q = 0$, yields the upper limit on $H'$, equation 9; the upper limit on $q$, $q = Q_{in}/\ell$, yields the lower limit on $H'$, equation 11.

The Hardy-Cross technique of pipe network analysis, employing the method of balancing heads\(^1\), can be adapted to systems having lines with distributed take-off, such as that illustrated in Figure 2.

FIGURE 2. DISTRIBUTION SYSTEM INCLUDING LINES WITH DISTRIBUTED TAKE-OFF.
In this method, the flows, \( Q \), entering and leaving the system and the distributed take-offs, \( q \), are assumed to be known from field data. Flows are initially assumed in each loop such that \( \Sigma Q = 0 \) at each junction. These flows do not necessarily yield head losses such that \( \Sigma H = 0 \) around a loop. In each loop a small counterflow, \( \Delta Q \), is therefore introduced such that \( \Sigma H = 0 \) around the loop. Using equations 3 and 8,

\[
\Sigma H = \Sigma k (Q + \Delta Q)^n + \frac{1}{n+1} \sum \frac{k' i}{q} \left[ (Q_{in} + \Delta Q)^n - (Q_{out} + \Delta Q)^n \right] = 0
\]

where flow both in pipes having distributed take-off and in conventional pipes has been accounted for. Expanding each power series and neglecting terms involving \( \Delta Q^2 \) and higher order quantities,

\[
\begin{align*}
\Sigma kQ^n + n \Sigma k (\Delta Q)Q^{n-1} + \ldots & \\
+ \frac{1}{n+1} \sum \frac{k'}{q} Q_{in}^{n+1} + \sum \frac{k'}{q} (\Delta Q)Q_{in}^n + \ldots & \\
- \frac{1}{n+1} \sum \frac{k'}{q} Q_{out}^{n+1} - \sum \frac{k'}{q} (\Delta Q)Q_{out}^n - \ldots & = 0
\end{align*}
\]

Solving for \( \Delta Q \),

\[
\Delta Q = -\frac{\Sigma kQ^n + \frac{1}{n+1} \sum \frac{k'}{q} \left( Q_{in}^{n+1} - Q_{out}^n \right)}{n \Sigma kQ^{n-1} + \sum \frac{k'}{q} (Q_{in}^n - Q_{out}^n)} \quad \ldots 12
\]

or, making use of equations 3 and 9,

\[
\Delta Q = -\frac{\Sigma H + \frac{\Sigma H'}{Q}}{n \Sigma \frac{H}{Q} + \Sigma H''} \quad \ldots 13
\]
where

\[ \Sigma H'' = \varepsilon \frac{k''}{q} \left( Q_{\text{in}}^n - Q_{\text{out}}^n \right) \] ...

This term has no physical meaning whereas the other terms in equation 13 do.

As in the traditional Hardy-Cross method, the corrections, \( \Delta Q \), yield new values of flow for the various pipes in the network. The procedure continues in an iterative fashion until the desired degree of accuracy is achieved, the result being the correct value of flow into and out of each junction as well as the head loss in each pipe.

Hazen-Williams Equation

The Hazen-Williams equation\(^2\) is an example of the exponential relationship between head loss and flow in a pipe given by equation 3

\[ H = 1.5 \times 10^{-5} \left( \frac{Q}{C} \right)^{1.85} \frac{k}{d^{4.87}} \] ...

where \( k \) of equation 3 is

\[ k = \frac{1.5 \times 10^{-5} k}{C \frac{1.85}{d^{4.87}}} \] ...

The following units have been used

- \( H \) = head loss (ft)
- \( Q \) = flow (gal/day)
- \( \ell \) = length (ft)
- \( d \) = diameter (in)
- \( C \) = coefficient of hydraulic capacity (determined from field tests)

\(^2\)ibid, p. 12-8.
The Hazen-Williams approach can be used in other previously derived equations by noting that
\[ k' = k = \frac{1.5 \times 10^{-5}}{c^{1.85} \cdot d^{4.87}} \]
and setting \( n \) equal to 1.85.

APPLICATIONS
Loss Detection

Given: Pipe having distributed take-off and
\[ \ell = 1,000 \text{ ft.} \]
\[ d = 3 \text{ in.} \]
\[ Q_{in} = 100,000 \text{ gal/day} \]
\[ C = 100 \]

Find: \( q \) for a pressure drop of 8 psi over the length of the pipe.

Solution: \( q \) can be determined from equation 7 by means of the iterative procedure presented in the flow diagram in Appendix 1. A Fortran computer program which utilizes this procedure is also given in Appendix 1. The answer to this problem is \( q = 32.0 \text{ gal/day/ft.} \)

Distribution System Analysis

Given: Network shown in Figure 3 and \( C = 100 \).

Find: Flow and pressure distribution within the network.

Solution: With the aid of equation 13, the Hardy-Cross iterative technique can be used to solve this problem. A Fortran computer
FIGURE 3. EXAMPLE PROBLEM: DISTRIBUTION SYSTEM ANALYSIS.
program which utilizes this technique is given in Appendix 2. The following flows were assumed:

\[
Q_1 = 175,000 \text{ gal/day}
\]

\[
Q_2 = -25,000 \text{ gal/day}
\]

\[
Q_3 = 75,000 \text{ gal/day}
\]

\[
Q_4 \text{ in} = 75,000 \text{ gal/day}, \quad Q_4 \text{ out} = 25,000 \text{ gal/day}
\]

where the subscript denotes pipe number and clockwise flows are positive. Five iterations yield a correction, \(\Delta Q\), of less than 10 gal/day. The rate of convergence is quite rapid even though the assumed flows were very different from the final ones. The results of the five iterations are given in Table 1. The final flows and head losses are:

\[
Q_1 = 32,700 \text{ gal/day}, \quad H_1 = 23.0 \text{ ft}
\]

\[
Q_2 = -167,000 \text{ gal/day}, \quad H_2 = -8.07 \text{ ft}
\]

\[
Q_3 = -67,300 \text{ gal/day}, \quad H_3 = -12.2 \text{ ft}
\]

\[
Q_4 = -67,300 \text{ gal/day}, \quad Q_{\text{out}} = -117,000 \text{ gal/day}, \quad H_4 = -2.74 \text{ ft}
\]

The final heads at each junction are:

\[
H_D = 200 \text{ ft (86.7 psig)}
\]

\[
H_A = 200-8.07 = 192 \text{ ft (83.2 psig)}
\]

\[
H_B = 192-2.74 = 189 \text{ ft (81.9 psig)}
\]

\[
H_C = 189-12.2 = 177 \text{ ft (76.7 psig)}
\]

or, \(H_C = 200-23.0 = 177 \text{ ft (76.7 psig)}\).
**TABLE 1. RESULTS OF HARDY-CROSS ITERATIVE TECHNIQUE FOR DISTRIBUTION SYSTEM ANALYSIS PROBLEM.**

<table>
<thead>
<tr>
<th>ITERATION NO.</th>
<th>PIPE NO</th>
<th>LENGTH</th>
<th>DIAMETER</th>
<th>Q</th>
<th>QDI</th>
<th>QDD</th>
<th>H</th>
<th>H/Q</th>
<th>H^t</th>
<th>H^T^t</th>
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DQ = -0.935E+05

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DQ = -0.411E+05

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DQ = -0.993E+04
**TABLE 1. RESULTS OF HARDY-CROSS ITERATIVE TECHNIQUE FOR DISTRIBUTION SYSTEM ANALYSIS PROBLEM (CONTINUED).**

**ITERATION 47, 4**

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**DQ = -0.756E+03**

**ITERATION 47, 5**

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<td>0.000E+00</td>
<td>0.703E-03</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
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<tr>
<td>2</td>
<td>0.500E+03</td>
<td>0.400E+01</td>
<td>-0.167E+36</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>-0.850E-01</td>
<td>0.435E-04</td>
<td>0.935E+00</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>3</td>
<td>0.100E+04</td>
<td>0.300E+01</td>
<td>-0.673E+35</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>-0.122E+02</td>
<td>0.181E-03</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
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<tr>
<td>4</td>
<td>0.500E+03</td>
<td>0.400E+01</td>
<td>0.000E+00</td>
<td>-0.673E+05</td>
<td>-0.117E+06</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>-0.270E+01</td>
<td>0.537E-04</td>
</tr>
</tbody>
</table>

**DQ = -0.457E+01**
Distribution System Modification

Given: Network shown in Figure 4 and C = 100 (note that this network is the same as that of the previous problem except that a distributed take-off has been added to pipe 3 and the input flow increased accordingly).

Find: Flow and pressure distribution within the network.

Solution: The method of solution is the same as that of the previous problem. The following flows were assumed:

\[ Q_1 = 50,000 \text{ gal/day} \]
\[ Q_2 = -250,000 \text{ gal/day} \]
\[ Q_3 \text{ in} = -50,000 \text{ gal/day}, Q_3 \text{ out} = -150,000 \text{ gal/day} \]
\[ Q_4 \text{ in} = -150,000 \text{ gal/day}, Q_4 \text{ out} = -200,000 \text{ gal/day} \]

In this case, because the assumed flows closely approximate the final ones, only two iterations are required to achieve a correction, \( \Delta Q \), of less than 10 gal/day. The results of the two iterations are given in Table 2. The final flows and head losses are:

\[ Q_1 = 50,900 \text{ gal/day}, H_1 = 52.1 \text{ ft} \]
\[ Q_2 = -249,000 \text{ gal/day}, H_2 = -16.8 \text{ ft} \]
\[ Q_3 \text{ in} = -49,100 \text{ gal/day}, Q_3 \text{ out} = -149,000 \text{ gal/day}, H_3' = -26.5 \text{ ft} \]
\[ Q_4 \text{ in} = -149,000 \text{ gal/day}, Q_4 \text{ out} = -199,000 \text{ gal/day}, H_4' = -8.73 \text{ ft} \]

The final heads at each junction are:

\[ H_D = 200 \text{ ft (86.7 psig)} \]
\[ H_A = 200 - 16.8 = 183 \text{ ft (79.3 psig)} \]
FIGURE 4. EXAMPLE PROBLEM: DISTRIBUTION SYSTEM MODIFICATION.
\[ H_B = 183 - 8.73 = 174.5 \text{ ft (75.6 psig)} \]

\[ H_C = 174.5 - 26.5 = 148 \text{ ft (64.1 psig)} \]

or, \[ H_C = 200 - 52.1 = 148 \text{ ft (64.1 psig)} \]

Note that, although the head at junction C is still adequate for household use, it has decreased significantly because of the increased flow requirements in the network.
TABLE 2. RESULTS OF HARDY-CROSS ITERATIVE TECHNIQUE FOR DISTRIBUTION SYSTEM MODIFICATION PROBLEM.

ITERATION NO. 1

<table>
<thead>
<tr>
<th>PIPE NO</th>
<th>LENGTH</th>
<th>DIAMETER</th>
<th>Q</th>
<th>QDI</th>
<th>QOD</th>
<th>H</th>
<th>H/Q</th>
<th>H#</th>
<th>H##</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.100E+04</td>
<td>0.200E+01</td>
<td>0.500E+05</td>
<td>0.000E+00</td>
<td>0.500E+00</td>
<td>0.100E-02</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>2</td>
<td>0.500E+03</td>
<td>0.400E+01</td>
<td>-0.250E+06</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>-0.170E+12</td>
<td>0.678E-04</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>3</td>
<td>0.100E+04</td>
<td>0.300E+01</td>
<td>0.000E+00</td>
<td>-0.500E+05</td>
<td>-0.150E+06</td>
<td>0.000E+00</td>
<td>-0.297E+12</td>
<td>0.465E-03</td>
<td>0.465E-03</td>
</tr>
<tr>
<td>4</td>
<td>0.500E+03</td>
<td>0.400E+01</td>
<td>0.000E+00</td>
<td>-0.150E+06</td>
<td>-0.200E+06</td>
<td>0.000E+00</td>
<td>-0.881E+12</td>
<td>0.726E-03</td>
<td>0.726E-03</td>
</tr>
</tbody>
</table>

DQ = 0.352E+03

ITERATION NO. 2

<table>
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<tr>
<th>PIPE NO</th>
<th>LENGTH</th>
<th>DIAMETER</th>
<th>Q</th>
<th>QDI</th>
<th>QOD</th>
<th>H</th>
<th>H/Q</th>
<th>H#</th>
<th>H##</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.100E+04</td>
<td>0.200E+01</td>
<td>0.500E+05</td>
<td>0.000E+00</td>
<td>0.300E+00</td>
<td>0.521E+02</td>
<td>0.102E-02</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>2</td>
<td>0.500E+03</td>
<td>0.400E+01</td>
<td>-0.249E+06</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>-0.169E+02</td>
<td>0.676E-04</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
</tr>
<tr>
<td>3</td>
<td>0.100E+04</td>
<td>0.300E+01</td>
<td>0.000E+00</td>
<td>-0.491E+05</td>
<td>-0.149E+06</td>
<td>0.000E+00</td>
<td>-0.269E+12</td>
<td>0.461E-03</td>
<td>0.461E-03</td>
</tr>
<tr>
<td>4</td>
<td>0.500E+03</td>
<td>0.400E+01</td>
<td>0.000E+00</td>
<td>-0.149E+06</td>
<td>-0.199E+06</td>
<td>0.000E+00</td>
<td>-0.875E+12</td>
<td>0.927E-03</td>
<td>0.927E-03</td>
</tr>
</tbody>
</table>

DQ = -0.399E+01
Flow Diagram for Solution of Loss Detection Problem

1. Read: \( c, d, Q_{in}, \xi, \Delta p, n \)

2. \( q_{max} = Q_{in}/\xi \)
   \( q_{min} = 0 \)

3. \( H' = \Delta p \left( \frac{144}{62.4} \right) \)  

4. \( k' = \frac{1.5 \times 10^{-5}}{c^n d^4.67} \)

5. \( q = q_{max} - q_{min} \)

6. \( H'_{test} = k' \left[ q_{in}^{n+1} - (Q_{in} - q^n)^{n+1} \right] \)  
   \( (n+1) \ Q \)

7. \( \delta = H'_{test} - H' \)

8. **IF** \( |\delta| \leq 0.1 \)
   **Yes** Write: \( q \)
   **No**

9. **IF** \( \delta > 0 \)
   \( q_{min} = q \)
   **Write**: \( q \)

10. **IF** \( \delta < 0 \)
    \( q_{max} = q \)
    **Write**: \( q \)

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**Fortran Computer Program for Loss Detection**

```fortran
REAL L,N,KP
READ(9,1)L,D,QIN,C,DELTAP,N
1 FORMAT(6F10.0)
QMAX=QIN/L
QMIN=0.
HP=DELTAP*144./62.4
KP=1.56-S/(C*N*D**4.87)
Q=(QMAX+QMIN)/2.
HPTEST=KP*(QIN**2*(N+1.0)-(QIN-Q*L)**2*(N+1.0))/((N+L.)*Q)
DELTA=HPTEST-HP
IF(ABS(DELTA).LE.0.1)GO TO 2
IF(DELTA).LE.0.1)GO TO 2
4 QMAX=Q
GO TO 6
5 QMIN=Q
GO TO 6
7 WRITE(3,3)Q
3 FORMAT(/1X,'Q=',F5.1)
STOP
END
```
APPENDIX 2

Fortran Computer Program for Distribution System Analysis

```
DIMENSION J(1), AL(I), H(I), QD(I), QD2(I), HP(I), SHPD(I)

READ (9,13) (J(I),I=1,1)
READ (9,14) (AL(I),I=1,1)
READ (9,15) (H(I),I=1,1)
READ (9,16) (QD(I),I=1,1)
READ (9,17) (QD2(I),I=1,1)
READ (9,18) (HP(I),I=1,1)
READ (9,19) (SHPD(I),I=1,1)

C=100.
&1.5E-05
&=1.45
CEN=0.0

M=700. L=1.13
SHO=0.
SHD=0.
SHPD=0.

WRITE (3,50)

FORMAT(7H1 ITERATION 50, I3)
WRITE(3,60)
FORMAT(11H1 PIPE 7 LENGTH DIAMETER 7 QD1
1 1 7 9 D QD D H M
1.5 4
1.4 4
500 J=1.4
1.5 4
IF (10(J)>100,700,13)
100 AK=AL(J)/CEN(J)**4.37
110 QH=AK*QD(J)**4.7
120 GY=0.001
130 H=H(J)-G Y
140 QD=QD(J)/H4.7
150 QD=QD(J)/H4.7
160 QD=QD(J)/H4.7
170 GY=GY+1.0
180 IF (GY>100,7,7)
200 IF (QD(J)>100.23,7,7)
210 MP=MHP(J)**2.7
220 IF (QD(J)>2.33,7,7)
230 HP=H(J)**1.23.45
240 GY=GY+1.0
250 H=H(J)-G Y
260 MP=MHP(J)**2.7
270 GY=GY+1.0
280 IF (GY>100,7,7)
290 SHPD=SHPD(J)
300 CONTINUE
310 WRITE(1,30) (J(I),I=1,1), (SHPD(I),I=1,1)
320 FORMAT(16I9.E12.1)
330 CONTINUE
340 WRITE(3,600)
600 FORMAT(/10I9.3)
620 GY=GY+1.0
630 IF (GY>100.23,7,7)
640 GO TO 700
650 CONTINUE
660 CONTINUE
700 STOP
END
```